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GEOMETRY.

Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

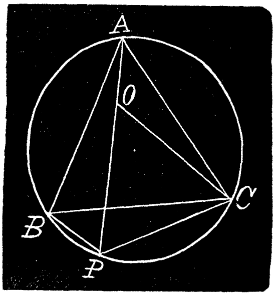
SOLUTIONS TO PROBLEMS.

19. Proposed by J. A. CALDERHEAD, Superintendent of Schools, Limaville, Ohio.

If any point be taken in the circumference of a circle, and lines be drawn from it to the three angles of an inscribed equilateral triangle, prove that the middle line so drawn is equal to the sum of the other two.

I. Solution by CHARLES E. MYERS, Canton, Ohio; P. S. BERG, Apple Creek, Ohio; Professor F. E. MILLER, Westerville, Ohio; R. H. YOUNG, Sunbury, Pennsylvania.

Let ABC be an equilateral triangle inscribed in a circle, P any point in the circumference; and PB, PC and PA lines drawn from the point to the three angles.



On PA take $PO=PC$, and join OC. The angles OPC and OPB, being measured by equal arcs, are equal and constant $=60°$, and since $PO=PC$, the the triangle POC is equilateral and $OC=PC$.

In the triangles OCA and BCP we have $OC=PC$; $AC=BC$ and the angle ACO=angle BCP; therefore $AO=BP$. But $OP=PC$; therefore $BP+PC=PO+OA=PA$.

II. Solution by J. W. WATSON, Middle Creek, Ohio.

Let ABC be the inscribed equilateral triangle. P any point in the circumference. Join AP, CP and BP. We are to prove $AP=BP+CP$.

Put $a=$ a side of the triangle, $n=AP$, $z=BP$ and $s=CP$.

In the triangle APB, $a^2=n^2+z^2-nz$ since the angle APB is $60°$.

In the triangle CPB, $a^2=s^2+z^2+sz$ since the angle CPB is $120°$.

$\therefore n^2+z^2-nz=s^2+z^2+sz$, $n^2-s^2=sz+nz$ or $(n-s)(n+s)=z(n+s)$, $n-s=z$. $\therefore n=s+z$, or $AP=CP+BP$.

III. Solution by Professor J. R. BALDWIN, A. M., Davenport, Iowa, and T. A. SIMMONS, St. Mary's Kentucky.

Let P be any point in the circumference, ABC the inscribed equilateral triangle of which A and C lie next to P.

Draw AP and PC.

By a well known proposition $AB\times PC+AP\times BC=AC\times PB$, (1).

But $AB=BC=AC$, hence casting out the equal factor out of (1) we have $PC+AP=PB$. *Q. E. D.*

Also solved by *G. B. M. ZERR*, *J. H. DRUMMOND*, *M. A. GRUBER*, *J. K. ELLWOOD*, and *P. H. PHILBRICK*. ☞ Three other solutions *without names of contributors* were received.

21. Proposed by CHARLES E. MYERS, Canton, Ohio.

A cistern 6 feet in diameter contains 3 inches of water. If a cylinder, four feet long and one foot in diameter, be laid in a horizontal position on the bottom, to what height will the water rise?

Solution by Professor G. B. M. ZERR, Principal of High School, Staunton, Virginia.

Let 2θ be the angle at the centre subtended by the chord made by the surface of the water.

Now since radius=6 inches and length=48 inches, area segment=36 $(\theta-\sin\theta\cos\theta)$; volume$=48\times36(\theta-\sin\theta\cos\theta)$; height of water$=6(1-\cos\theta)$. Volume also equals amount of rise in water. Volume$=\{6(1-\cos\theta)-3\}\pi(36)^2$.

$\therefore\ 48\times36(\theta-\sin\theta\cos\theta)=\{6(1-\cos\theta-3\}\pi(36)^2$, $\therefore\ 4\theta-4\sin\theta\cos\theta=9\pi-18\pi\cos\theta$.

Now by double position, $\theta=63^\circ\ 13'\ 55.4''$.

$\therefore\ 6(1-\cos\theta)=3.29772$.

$\therefore$ water rises $3.29772-3=.29772$ inches.

Also solved by *SETH PRATT*, and *H. C. WHITAKER*.

22. Proposed by J. A. TIMMONS, St. Mary's, Kentucky.

Given, the perimeter of a triangle$=100(2s)$, the radius of the inscribed circle $=9(r)$, and the radius of the circumscribed circle$=(R)$; it is required to find (1) the sides of the triangle, (2) the radius of the circle circumscribing the triangle formed by bisecting the exterior angles of the original triangle, (3) the area of the triangle thus formed: all in terms of R, r, s.

I Solution by H. C. WHITAKER, M. S., M. E., Professor of Mathematics, Manual Training School, Philadelphia, Pennsylvania.

From the properties of the triangle,

$$a+b+c=2s$$
$$ab+ac+bc=s^2+r^2+4Rr$$
$$abc=4Rrs,$$

whence from the theory of equations, these quantities are the co-efficients of a cubic equation, the roots of which are the three sides. The numerical values are 32, 28.43225 and 39.56775.

Second. The length of the side (passing through A) of the outer triangle is $\dfrac{b\cos\frac{1}{2}C}{\cos\frac{1}{2}B}+\dfrac{c\cos\frac{1}{2}B}{\cos\frac{1}{2}C}$, its opposite angle being $\dfrac{B}{2}+\dfrac{C}{2}$. Dividing the side by twice the sine of this angle gives $\dfrac{b\cos^2\frac{1}{2}C+c\cos^2\frac{1}{2}B}{\sin B\cos^2\frac{1}{2}C+\sin C\cos^2\frac{1}{2}B}$ which by making $c=\dfrac{b\sin C}{\sin B}$ reduces to $b\div\sin B=2R$;—that is, the radius of the circle circumscribed about the outer triangle is twice the radius of the circle circumscribed about the inner triangle. The numerical value is 40.

Third. The length of the side passing through A of the outer triangle is also equal to $\dfrac{S}{\cos\frac{1}{2}B\cos\frac{1}{2}C}$ with similar expressions for the other sides. Now the area of any triangle equals the product of the three sides divided by four times the radius of the circumscribed circle; that is

$$\text{Area}=\frac{S^2}{8R\cos^2\frac{1}{2}A\cos^2\frac{1}{2}B\cos^2\frac{1}{2}C}=2Rs$$